This is a closed book, closed note exam. You may use your calculator. Show all work. Answers with insufficient supporting work will not receive full credit.

1. Find the general solution to the ode’s below.
   
   (a) \( \frac{dy}{dt} = ty \). 
   
   (b) \( \frac{dy}{dt} = 2ty - 5e^{t^2} \).
   
   (c) \( \frac{dy}{dt} = \frac{y}{1+t} - \frac{y}{t} + t^2(1 + t) \) use \( u = \frac{y}{1+t} \).

2. Describe the long-term behavior of the solution to the differential equation
   
   \[ \frac{dy}{dt} = y^2 - 7y + 3 \]
   
   with the initial condition
   
   (a) \( y(0) = 0 \) 
   
   (b) \( y(0) = 7 \) 
   
   (c) \( y(0) = 3 \)

3. A college professor is currently depositing $8000 per year into a retire-
   ment account paying 10% per year. After 30 years she retires, transfers
   the money accumulated into an account paying 8% per year and be-
   gins drawing $5000 per month from the account. With no additional
   deposits, how long will the money last? Bonus: Suppose that she wants
   to adjust her yearly income to account for inflation. For this purpose,
   we’ll assume an annual inflation rate of 4%. How long will her money
   last? Give your answer in feet. (joke, ha ha!)
4. Match the equation to the slope field. Explain your choices.

\[
\begin{align*}
\frac{dy}{dt} &= 1 - t^2 & \text{(1)} \\
\frac{dy}{dt} &= y^2 - 1 & \text{(2)} \\
\frac{dy}{dt} &= y^2 - t^2 & \text{(3)} \\
\frac{dy}{dt} &= t^2 - y^2 & \text{(4)} \\
\frac{dy}{dt} &= 1 - y^2 & \text{(5)} \\
\frac{dy}{dt} &= t^2 - 1 & \text{(6)} \\
\frac{dy}{dt} &= y^2 + 3y + 1 & \text{(7)}
\end{align*}
\]
My answer is:

My reasons are:

\[ x' = x^2 + 3x + 1 \]

The backward orbit from (0.19, 5.9)
Ready.
The forward orbit from (0.61, 5.9) left the computation window.
The backward orbit from (0.61, 5.9)
Ready.
My answer is:

My reasons are:

$x = x^2 - t^2$

The forward orbit from (0, 0.5) left the computation window.
The backward orbit from (0, 0.5) failed.
Computing the field elements.
Ready.
My answer is:
My reasons are:
5. Consider the population model

\[
\frac{dP}{dt} = \frac{-P^2}{50} + 2P
\]

for a species of fish in a lake. Suppose it is decide that fishing will be allowed, but it is unclear how many licenses should be issued. Suppose that the average fisherman catches 3 of the fish per year.

(a) What is the largest number of licenses that can be issued if the fish are to have a chance at survival? Explain.

(b) Suppose that the number of licenses from part a is issued, how does the behavior of the population depend on the initial population?

(c) Suppose that the population of fish is 50 and a careless clerk accidently issues 5 extra licenses for the year. How many licenses should be issued the next year? Explain.